Cavity quantum electrodynamics with semiconductor double-dot molecules on a chip

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We describe a coherent control technique for coupling electron spin states associated with semiconductor double-dot molecule to a microwave stripline resonator on a chip. We identify a novel regime of operation in which strong interaction between a molecule and a resonator can be achieved with minimal decoherence, reaching the so-called strong coupling regime of cavity QED. We describe potential applications of such a system, including low-noise coherent electrical control, fast QND measurements of spin states, and long-range spin coupling.

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Controlling quantum behavior of realistic solid-state systems is an intriguing challenge in modern science and engineering. While over the past several decades much progress has been made in manipulation of isolated atomic and optical systems, only recently have solid-state systems demonstrated controlled, coherent behavior at a single quantum level [1, 2, 3, 4, 5, 6]. The complex environment of solid state systems make them significantly more challenging to isolate and operate coherently. Furthermore, robust quantum control techniques analogous to those used in AMO physics still need to be developed.

Recently, a novel approach to quantum manipulation of spin-based quantum systems in semiconductor quantum dots has been proposed and experimentally realized [6, 7, 8]. This approach combines spin and charge manipulation to take advantage of the long memory times associated with spin states and, at the same time, to enable efficient readout and coherent manipulation of coupled spin states using intrinsic interactions. While this allows one to consider architectures based on pulsed quasi-static electrical control and static magnetic field [9], for many purposes it is desirable to develop fast and robust quantum control techniques based on microwave manipulation.

In this Letter we describe a technique for electrical coupling of electron spin states associated with semiconductor double-dot molecule to microwave stripline resonator on a chip [5, 10, 11]. The essential idea is to use an effective electric dipole moment associated with exchange coupled spin states of a double-dot molecule to couple to the oscillating voltage associated with a stripline resonator. Taking into account the main decoherence mechanisms for both spin and charge degrees of freedom, we identify an optimal point of operation in which strong interaction between molecule and resonator can be achieved with minimal decoherence, thereby enabling the strong coupling regime of cavity QED. Finally, we describe potential applications including low-noise coherent electrical control, fast QND measurements of spin states, and long-range coupling of pairs of spins.

Before proceeding we note the early proposals for achieving the strong coupling regime of cavity QED with

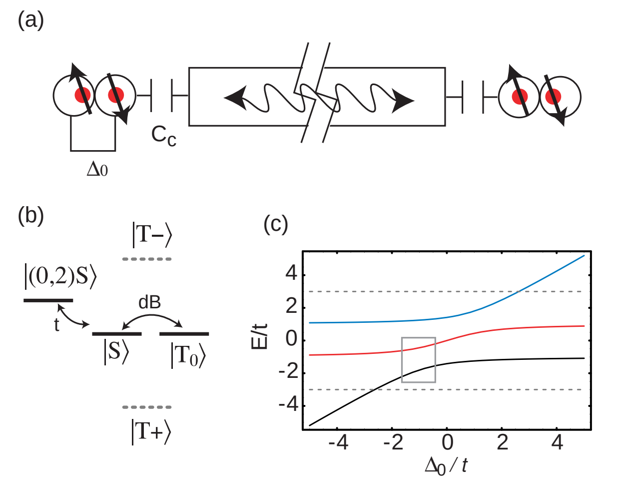

FIG. 1: (a) Schematic of two double dots, biased with external potential Δ_0, capacitively coupled to a transmission line resonator. (b) Energy level diagram showing the (0,2) singlet and the four (1,1) two-spin states. (c) Low energy spectrum in units of t with small gradient $dB = t/10$: for large, negative Δ_0, the ground state is $|(0,2)S\rangle$. The T_0 triplet (red) and $T_\pm$ triplets (dashed grey) are far from resonance with the $(0,2)$ triplet. Note the optimal point (gray box) occurs at the left-most avoided crossing.

Cooper pair qubits and single electrons in double-dot molecules [10, 11]. Strong coupling of superconducting qubits to stripline cavities have been recently implemented in pioneering experiments by Schoelkopf and coworkers [5]. This system has enabled a range of beautiful demonstrations ranging from control and measurement of the qubit through the resonator to SWAP of the qubit with the state of the resonator. Finally, ideas similar to the present work have been proposed recently, using vertical quantum dot systems [12].

We consider the specific system outlined in the Figure 1. Here two electron spins are localized in adjacent quantum dots, coupled via tunneling. One of the dots is capacitively coupled to a transmission line resonator. A modest (100 mT) external magnetic field along an axis z Zeeman-splits the spin-aligned states, $|T_+\rangle = |\uparrow\uparrow\rangle$ and $|T_-\rangle = |\downarrow\downarrow\rangle$, while the spin-anti-aligned states are used as

a qubit degree of freedom: $|(1,1)T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ and $|(1,1)S\rangle = (|\downarrow\uparrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. The notation (n_L, n_R) labels the number of electrons in the left and right quantum dots. In addition to the qubit states, an auxiliary singlet state with two electrons in one quantum dot, $|(0,2)S\rangle$, is coupled via tunneling t to the separated singlet, $|(1,1)S\rangle$ (Fig. 1b). The energy of the auxiliary state is determine largely by the bias Δ due to the electric field; control of Δ allows to control the qubit's evolution. The Hamiltonian for this three state system is:

$$\begin{aligned} H_{DD} &= \Delta|(0,2)S\rangle\langle(0,2)S| + t\,|(1,1)S\rangle\,\langle(0,2)S| + \\ & dB\,|(1,1)S\rangle\,\langle(1,1)T_0| + \text{H.c.}. \end{aligned} \tag{1}$$

We have introduced a static magnetic field gradient between the two dots $dB = g^* \mu_B (B_z^L - B_z^R)$, which mixes the singlet and triplet states. This system has been studied in detail in Refs. 8, 10, 13, 14. Parameter ranges for experiments are $t \simeq 0 - 10\mu$eV and $dB \sim 0.1 - 1\mu$eV. State preparation, measurement, 1-qubit gates, and local 2-qubit gates can be achieved by changing the bias Δ between $|(0,2)S\rangle$ and $|(1,1)S\rangle$ [6, 7, 9].

Interaction with the resonator is included naturally by writing the bias as a contribution from static fields, Δ_0, set by voltages on the gates defining the double dot, and a contribution from the resonator itself:

$$\Delta = \Delta_0 + e\hat{V}\frac{C_c}{C_{\text{tot}}}\ , \tag{2}$$

where C_c is the capacitive coupling of the resonator to the dot, while C_{tot} is the total capacitance of the double dot. The voltage due to the resonator of length l, capacitance per unit length C_0, and impedance Z_0 is quantized as [10]

$$\hat{V} = \sum_k \sqrt{\frac{\hbar\omega_k}{lC_0}}(\hat{a}_k + \hat{a}_k^\dagger)\ . \tag{3}$$

We now deduce an effective Hamiltonian for the system when the splitting between eigenstates of Eq. 1, ω, is comparable to the fundamental mode frequency of the resonator, $\omega_0 = \pi/lZ_0C_0$ (i.e., the cavity detuning $\delta = \omega - \omega_0$ is small). Neglecting the higher energy modes, we write the Hamiltonian for the resonator itself as $H_{TLR} = \hbar\omega_0\hat{a}^\dagger\hat{a}$, where $\hat{a}$ is the lowest energy mode destruction operator, and the interaction with the dot is

$$U = g(\hat{a} + \hat{a}^\dagger)|(0,2)S\rangle\langle(0,2)S| \tag{4}$$

where $g = e\frac{C_c}{C_{\text{tot}}lC_0}\sqrt{\hbar\pi/Z_0} = \eta\omega_0$ is the vacuum Rabi coupling between the double dot and the resonator. In practice, for $\omega_0 = 2\pi \times 10$ GHz, $g = 2\pi \times 100$ MHz (i.e., $\eta = 10^{-2}$) is achievable [5, 10, 11]). A reduction of ω_0 by lowering lC_0 results in a comparable reduction of g. Finally, as $Q > 10^6$ resonators are possible in the microwave domain [5, 10], the quantization of the resonator voltage is appropriate. In practice, the need to work with a finite local external magnetic field (100 mT) may limit $Q \approx 10^4$ [15].

We seek a set of parameters $\{\Delta_0, T_0, dB\}$ such that the system can be coupled to and controlled by the resonator. This "operating point" should maximize the coupling to the resonator and minimize the noise in the combined system. To motivate the optimal choice, let us first proceed with small tunnel coupling t and cavity coupling g. In this case the eigenstates are $|\downarrow\uparrow\rangle, |(0,2)S\rangle, |\uparrow\downarrow\rangle$, with eigenenergies $-dB, \Delta_0, dB$. Setting the energy difference $\tilde{\Delta} = dB - \Delta_0 \approx 0$, degenerate perturbation theory in the tunnel coupling t reveals an avoided crossing at this balanced point between $|\downarrow\uparrow\rangle$ and $|(0,2)S\rangle$ with an energy gap $\omega = \sqrt{\tilde{\Delta}^2 + 4t^2}$ around $\tilde{\Delta} = 0$ (Fig. 1b) and mixing angle $\theta = \frac{1}{2}\tan^{-1}(2t/\tilde{\Delta})$. Working in the rotating frame with the rotating wave approximation, we find an effective Hamiltonian for the combined system to be

$$H = \hbar(\omega - \omega_0)|1\rangle\langle 1| + g_{\text{eff}}\hat{a}\,|1\rangle\,\langle 0| + \text{H.c.} \tag{5}$$

where, at $\theta = \pi/4$, $|0(1)\rangle = (|\downarrow\uparrow\rangle - (+)\,|(0,2)S\rangle)/\sqrt{2}$ and

$$g_{\text{eff}} = -\frac{1}{2}\eta\omega_0 \sin 2\theta \approx -\eta t\ . \tag{6}$$

We choose $\theta = \pi/4$ for two reasons. First, at this point the gap ω is insensitive to first-order changes in Δ_0 and dB. Such zero-derivative points are optimal for controlling quantum bits with imperfect electronics and in the presence of low frequency noise [3, 16, 17, 18, 19]. In our case this suppresses fluctuations in control electronics (Δ_0) and in the magnetic field gradient (due to hyperfine interactions), which are the dominant dephasing terms. Second, the coupling coefficient g_{eff} is directly proportional to the energy splitting in the system, as would be expected of a "bare" coupling of comparable strength.

The cost of working at this point is increased sensitivity to fluctuations in t (it effects both g_{eff} and ω at first order) and a higher probability of relaxation due to the enhanced charge admixture of the qubit states (see the analysis of inelastic effects below). The former could be mitigated: as no time-dependent control of t is required, the gates that set t can be heavily filtered to greatly reduce potential noise.

For long term storage of the quantum information, we can adiabatically map the states $|0\rangle, |1\rangle$ to the spin states $|\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle$ via change of Δ_0, going to a well separated regime. As long as the change of Δ_0 is slow with respect to the splitting dB, the process is entirely adiabatic; furthermore, we are only sensitive to charge fluctuations on a frequency scale faster than $\sim dB$. The spin states, protected by occassional refocusing, are mostly insensitive to charge relaxation and dephasing [20, 21]. Thus, when long quantum memory times or insensitivity to charge-based relaxation is necessary, the system can be adiabatically mapped to such a separated regime; when operation

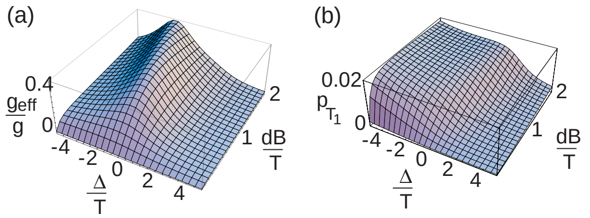

FIG. 2: (a) Coupling strength, $g_{\rm eff}$, in units of bare coupling g, as a function of field gradient dB and bias Δ_0. (b) Calculated probability of error in swapping a double-dot state with the resonator state due to charge-based relaxation using $t = 4$ μeV and an inverse temperature of 10 μeV.

with the resonator is necessary, they can be mapped back to the balanced regime of $\Delta_0 \approx -dB$.

We now proceed to generalize the above analysis to an arbitrary set of parameters $p = \{dB, t, \Delta_0\}$, treating the interaction with the cavity as a perturbation. We write the eigenenergies of H_{DD} as $E_m(p)$ ($m = 0, 1, 2$, and $E_0 \leq E_1 \leq E_2$) and the eigenvectors as

$$\begin{pmatrix} |0\rangle \\ |1\rangle \\ |2\rangle \end{pmatrix} = \chi^\dagger(p) \begin{pmatrix} |(1,1)T_0\rangle \\ |(1,1)S\rangle \\ |(0,2)S\rangle \end{pmatrix} \tag{7}$$

where χ is a 3x3 unitary matrix. In the rotating wave approximation, we may neglect all cavity interaction terms diagonal in the eigenstates. Furthermore, when the energy differences are non-degenerate ($E_1 - E_0 \neq E_2 - E_1$), we may also only keep coupling to the transition with energy $E_1 - E_0 \approx \hbar\omega_0$. Then,

$$U = g\hat{a}^\dagger \chi_{02}\chi_{21}^* |0\rangle \langle 1| + \text{H.c.} \tag{8}$$

Thus, $g_{\rm eff}(p) = g\chi_{02}(p)\chi_{21}^*(p)$. Keeping t fixed, we plot $g_{\rm eff}$ and the expected error in operations due to relaxation (calculated below). Of immediate interest is the regime of $dB \sim t$, where $g_{\rm eff}/g > 0.3$ and $\xi \neq 0$. This suggests that maximal coupling to the cavity occurs when $t, -\Delta_0$, and dB are all comparable.

We now demonstrate the favorable noise properties of the balanced system. Three kinds of error are considered: relaxation of the charge system in a time T_1, additional dephasing of the charge system in a time $T_{2,a} = T_2 - 2T_1$, and decay of the cavity photon at a rate κ. We assume the additional dephasing arises from low frequency fluctuations [3, 8, 9, 17, 19, 22], due to changes of the electrostatic gates and magnetic field gradient, e.g., as noise on Δ_0 and dB.

For charge-based relaxation, coupling to a phonon bath with spectral density $\rho(\omega) = \sum_k |g_k|^2\delta(\omega - \omega_k)$ will lead to decay [23]. Using the spin-boson model in the perturbative regime yields an overall error rate from relaxation and incoherent excitation:

$$1/T_1 = 2\pi\rho(\omega_{10})|\chi_{20}\chi_{12}|^2 \coth(\omega_{10}\beta/2) . \tag{9}$$

We estimate $\rho(\omega)$ at $\omega = 76\mu\text{eV}/\hbar$ using measured charge relaxation times [24, 27]. For a double quantum dot system in GaAs and small ω, $\rho(\omega) \propto \omega^3$ [13]. This indicates $2\pi\rho(\omega) \approx \hbar^2\omega^3/(1\text{meV})^2$ for the low energy limit. Reducing the resonant frequency lowers relaxation rates. Near the optimal point, with $\omega_{10} \sim 2\pi\times$ 2 GHz, $T_1 \sim 1\mu$s.

Additional dephasing ($T_{2,a}$ term) arises from variations of the energy gap, $\Delta_0(t) = \Delta_0 + \epsilon(t)$ with $\langle\epsilon(t)\epsilon(t')\rangle = \int d\omega\ S(\omega)e^{i\omega(t-t')}$. We assume a high frequency cutoff of the noise at $\gamma \ll E_1 - E_0, \omega_0$. The contribution to error at first order arises from $\omega_{10}(t) = \omega_{10} + \epsilon(t)(\partial\omega_{10}/\partial\Delta_0)$. When the pre-factor $\eta_\Delta = |\partial\omega_{10}/\partial\Delta_0|^2$ is non zero, there is first-order dephasing. In the rotating frame, the off-diagonal density matrix element evolves as

$$\rho_{01}(t) \sim \exp\left[-\eta_{\Delta_0}\int d\omega\ S(\omega)\frac{\sin^2(\omega\tau/2)}{(\omega/2)^2}\right] . \tag{10}$$

For example, when $\gamma \ll 1/\tau$, the decay is Gaussian with a time constant $T_2 \sim T_{2,\rm bare}/\sqrt{\eta_{\Delta_0}}$, where the characteristic time $T_{2,\rm bare} = 1/\sqrt{\int d\omega\ S(\omega)}$. Bare dephasing times of ~ 1 ns were observed for qubit frequencies of $\sim 2\pi\times 20$ GHz [24, 27]. However, longer $T_{2,\rm bare}$ times might be achieved for qubits at $\sim 2\pi \times 1$ GHz through better high- and low-frequency filtering of electronics noise. We will take $T_{2,\rm bare} = 10$ ns for the remainder of the paper; this limit arises from the combined effect of charge-based terms and nuclear spin-related dephasing [6, 25, 26].

At the zero derivative point ($\partial\omega/\partial\Delta_0 = 0$) second-order terms must be considered [3, 18, 28]. To the leading order [18], we find dephasing occurs over a timescale $T_2 \sim \omega_{10}(T_{2,\rm bare})^2$. More detailed calculations indicate that the underlying physics of the bath becomes important near zero-derivative points [18, 28]. Thus the optimal point combines the maximal coupling $g_{\rm eff}$ to the resonator with an extended T_2 time (Fig. 3). In this regime, with $\omega_0 = 2\pi \times 1.5$ GHz, $g_{\rm eff}T_2 \approx 100$. We note that away from the optimal point, even assuming maximal coupling, $g_{\rm eff}T_2 \lesssim 1$. For example, in the strongly biased regime ($\Delta/t \gg 1$), $g_{\rm eff}T_2 \approx gT_{2,\rm bare} \lesssim 1$. We conclude that only near the optimal point may the strong coupling regime be achieved.

We now discuss potential applications. Coherent control of the system with little sensitivity to low frequency noise is achieved by driving the resonator on resonance with a microwave pulse. We can describe this scenario by making the cavity state a coherent state, $|\alpha\rangle$, letting us replace $\hat{a}$ with α. We would expect Rabi oscillations between $|0\rangle$ and $|1\rangle$ with Rabi frequency $\Omega = g_{\rm eff}\alpha$. Low frequency components of the driving field (DC offsets) are greatly reduced because of the gap induced by t and the insensitivity of the gap energy ω on variations of Δ.

Another technique is a quantum non-demolition measurement. When the microwave transition is detuned from the cavity by $\tilde{\Delta}$, the evolution yeilds a qubit-dependent phase of $\phi_{QND} = \pm g^2\tau/\tilde{\Delta}$ (for $|0\rangle, |1\rangle$ states

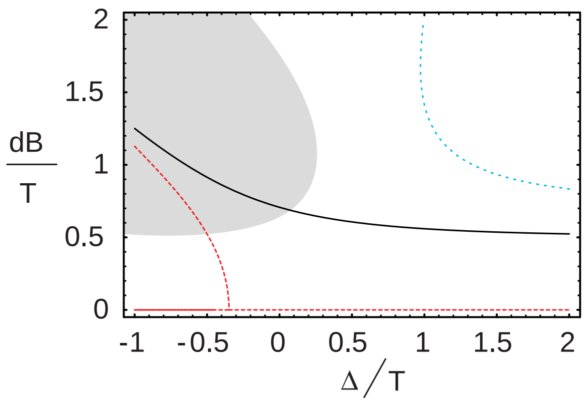

FIG. 3: Solutions of $\partial\omega_{10}/\partial\Delta_0 = 0$ (black), $\partial\omega_{10}/\partial t = 0$ (blue), and $\partial\omega_{10}/\partial dB = 0$ (red). The region of $g_{\rm eff}/g > 0.3$ is marked in grey.

respectively) for an injected microwave field over a time τ (limited by qubit decay). Assuming that the noise level of the detector exceeds the background by about a factor of ten, the fidelity of such a QND measurement of the encoded spin qubits is given by $F = 1 - 10\kappa/g^2 T_1$ [11]. This single qubit QND measurement can be completed in a time $1/\kappa \sim 1$ μs.

In addition, a wide variety of cavity QED quantum control techniques may be accessible. As an example, we focus on the SWAP operation, where the qubit state is "swapped" with a photonic state of the resonator. When there are no photons in the cavity and $\delta = 0$, the state $|0\rangle\,|0\rangle_{\rm cav}$ is stationary, while $|1\rangle\,|0\rangle_{\rm cav}$ oscillates with Rabi frequency $g_{\rm eff}/\hbar$ to the state $|0\rangle\,|1\rangle_{\rm cav})$ (we use $|n\rangle_{\rm cav}$ to indicate n photons in the cavity mode). When the time spent oscillating is $\pi\hbar/g_{\rm eff}$, a quantum state of the singlet–triplet system is mapped to the existence or absence of a cavity photon:

$$(\alpha\,|0\rangle + \beta\,|1\rangle)\,|0\rangle_{\rm cav} \to |0\rangle\,(\alpha\,|0\rangle + i\beta\,|1\rangle)_{\rm cav}\ . \qquad (11)$$

This process can be controlled by rapidly changing δ to and from zero. In essence, we can convert one quantum bit (the double-dot system) to another quantum bit (the cavity photon), which can then be used for a variety of quantum information tasks, such as long-distance quantum gates, quantum communication, and quantum repeaters [29, 30, 31]. Furthermore, it may allow for coupling to other qubit systems, such as atoms, molecules [32], or superconducting qubits [11].

As an example, we now detail the expected errors for the SWAP operation. Both T_1 and κ contribute to the decay of the system, which lead to population of the state $|0\rangle\,|0\rangle$, while dephasing, T_2, leads to phase errors in the transformation. Including only relaxation terms, yields a probability of error in "SWAP" of $p_{T_1} = \pi\frac{\kappa + 1/T_1}{g_{\rm eff}}$. In addition, pure dephasing (T_2) leads to an error with probability $p_{T_2} = \pi^2/(T_2 g_{\rm eff})^2$. The optimal point has $g_{\rm eff} \approx 0.4\eta E_{10}$, $1/T_1 \approx 0.2\Gamma(E_{10})$, and $T_{2,a} \approx \omega_{10} T_{2,\rm bare}^2$ from noise in Δ_0 and dB. Optimizing against both noise sources suggests that for a bare dephasing time of $T_{2,\rm bare} = 10$ ns, a cavity frequency of ~ 1.5 GHz is optimal for SWAP operations, with a probability of error $\approx 2\%$.

In summary, we have shown how spin states of double quantum dots can be coupled directly to microwave resonators. An optimal operating point with maximum coupling to the resonator and minimum coupling to some sources of charge and spin noise is found. This suggests a variety of powerful quantum control techniques may become possible for such a system.

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